Optimal Mechanism Design with Aftermarket Competitions

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Abstract

This paper derives the optimal selling mechanism for an indivisible object to entrants with private production costs in the primary market when the winning entrant needs to engage in Cournot competition with an incumbent afterwards. While mechanism designers have full power in determining the mechanism in the primary market, they cannot control the production levels in the aftermarket Cournot competition directly. As a result, one important and non-conventional instrument for the designers is determining the optimal information structure in the aftermarket to influent its outcome. We find that, in the optimal mechanism, designers rank the entrants according to their virtual costs, and more importantly, fully reveal the winning entrant's private production cost to the incumbent. This has important implications for implementing the optimal mechanism via auctions in practice: while first-price auctions with a reserve price and the announcement of the winning bid are optimal when the entrants are *ex-ante* symmetric, English auctions can never be optimal. We also study the scenario where designers have more control over the aftermarket and find that they are strictly better off.

Keywords: Aftermarket, Auctions, Cournot Competition, Hidden Actions, Hidden Information, Mechanism Design.

JEL Classifications: C72, D44, D82, D83, L12

1 introduction

Sellers as mechanism designers in real life often face potential buyers who will engage in aftermarket competitions upon acquiring the objects. For example, after Deutsche Telekom AG acquires VoiceStream Wireless in 2001 for $35 billion and founded T-Mobile USA, Inc. in July, 2002, it had

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to compete with other nationwide telecoms such as AT&T Mobility later on. When an entrant purchases a franchise from McDonald's and operates as its franchisee, he/she has to compete with other fast food franchisees such as KFC's in the local area in the aftermarket. When a firm obtains a licence from a government to operate in a regulated industry, it needs to interact with other firms already in the industry. When a company wins a patent for cost reducing technology from an inventor, it still needs to operate in the industry afterwards.[1] How should the mechanism designers, i.e., VoiceStream Wireless, McDonald's, governments and inventors, sell their objects foreseeing the competitions that buyers face in the aftermarket? The is the question to be addressed in this paper.

A common observation is that although designers have strong power in the primary market in determining how to sell the object, they usually have imperfect control over the aftermarket competitions. For instance, it is unlikely that Voicestream Wireless or McDonald's could intervene AT&T Mobility's or KFC franchisee's business decisions in the aftermarket. So how would Voicestream Wireless or McDonald's achieve their objectives in this situation? The solution lies in the utilization of information: Although the designers cannot directly dictate all actions in the aftermarket, they can change aftermarket incentives by revealing (a part of) information obtained in the primary market, and thus influence the outcome in the aftermarket. For example, the purchase price of $35 billion contains VoiceStream Wireless' private information, and whether the price is made public can influence the competitors' beliefs. Similarly, when the potential entrants bid for a McDonald's franchise, the bids contain their private information. As a result, McDonald's can determine how much information to reveal about those bids. Different information revelation rules result in different information structure in the aftermarket, directly and indirectly affecting the mechanism designer's payoff. As such, one important and non-conventional instrument for the designers is determining the optimal information structure in the aftermarket to influent its outcome.

From the designer's point of view, she is facing a mechanism design problem with hidden information, hidden actions and multiple agents. Although revelation principle applies as shown in Myerson[24] in this general framework, solving the model is usually quite difficult. In our model, take McDonald's for example, hidden information arises because the entrants have private information regarding their production costs; hidden actions are due to the fact that McDonald's is not able to directly control all actions in the aftermarket competition. Finally, there are many entrants and KFC franchisee in the environment. The most important feature and a technical challenge for theory is that the information structure in the aftermarket is endogenously determined by the designer to influence the agents' hidden actions optimally to her advantage, which distinguishes our model from most existing literature.

Some studies in the principal-agent literature combine the hidden information and hidden action problems in a single agent setup.[2] However, the issue of multiple agents makes a significant difference. With a single agent, after the agent reports his private information, there is no need for

[1] The issue of aftermarket competition has attracted ample attentions in the auction literature., see Chattopadhyay and Chatterjee [6], Das Varma [7], Fan et.al [8], Goeree [10], Katzman and Rhodes-Kropf [15], and Scarpatetti and Wasser [27], all of which will be reviewed later.

[2] See Caillaud et. al [4], Laffont and Tirole [16], Lewis and Sappington [18], Riley [25], and Riordan and Sappington [26].

the principal to reveal it back to the agent. With multiple agents, how much information regarding the agents' reports to reveal back to the agents is no longer irrelevant since it affects the agents' beliefs about each other, and thus influences their hidden actions.

Several papers do allow multiple agents, but the issue of optimal information revelation is avoided. Laffont and Tirole [17], Lewis and Sappington [19], and McAfee and McMillan [20] characterize the optimal contract with risk neutral privately informed agents who later choose unobservable efforts. After the winning agent obtains the contract, only he exerts efforts. As a result, there is no need to control information revelation. McAfee and McMillan [21] consider the optimal design of team mechanisms when risk neutral agents have privately known abilities and individual efforts are not observable in the team production. Their main focus is to identify the circumstances where linear contracts can implement the pseudo optimal revenue as if the hidden action problem were not present. As a result, the strategic information revelation is beyond the scope of their paper. One of our findings show that the designer's rent extraction ability is strictly reduced if hidden action problem arises on top of the hidden information problem, even when the agents are risk neutral. This leads to an open question in their paper: what is the optimal contract when the pseudo optimal revenue cannot be achieved?

In this paper, we consider an environment where a mechanism designer (franchise company, government, inventor, etc.) decides on how to sell an object (franchise, licence, patent, etc.) to a few potential entrants with privately known production costs in the primary market. The winning entrant, if any, then competes with an incumbent in the aftermarket modeled as a Cournot competition.[3] We assume that the designer has full power in designing the selling mechanism in the primary market, but has imperfect control in the aftermarket. For instance, the designer has no control over the incumbent at all; she can neither collect money from the incumbent nor dictate its production level in the aftermarket. Regarding the controlling power of the designer over the winning entrant in the aftermarket, we consider two different scenarios: partial control and no control, depending on whether the designer can dictate a production level for the winning entrant.[4] For the franchise and licence cases, the partial control scenario is more applicable. For the Voicestream Wireless and patent cases, it is more reasonable to assume that Voicestream Wireless and the inventor have no control over AT&T Mobility and the winning company, and therefore, the no control scenario is more suitable.

We are able to explicitly characterize the optimal mechanisms under general conditions in both scenarios. In the no control scenario, the constructed optimal mechanism is deterministic. The designer allocates the object to the entrant with the lowest virtual production cost, given that it is lower than a cutoff. This cutoff is increasing in both the market size and the incumbent's production cost. In this optimal mechanism, the designer fully reveals the winning entrant's reported private cost to the incumbent, and this information can be transmitted through the winning entrant's monetary transfer. As a result, the incumbent infers exactly the winning entrant's production cost and the outcome in the aftermarket coincides with a standard Cournot competition under complete information. This has important implications for implementing the optimal mechanism

[3] When no potential entrant wins, the incumbent remains a monopoly.

[4] The reason why we call the first scenario partial control is that the designer can only make decisions for the winning entrant but not for the incumbent in the aftermarket.

in practice. First, when there is a single potential entrant, it is never optimal for the designer to make a take-it-or-leave-it offer to the entrant. Second, when there are multiple symmetric potential entrants, while the optimal mechanism can be implemented by a first-price auction with a reserve price and together with the announcement of the winning bid, it can never be implementable via English auctions. In the no control scenario, while most results are similar, the outcome in the aftermarket is the same as a *modified* Stackelberg competition under complete information with the winning entrant being the leader and the incumbent being the follower. Entry happens less often and the designer achieves a strictly less revenue in the no control scenario than in the partial control scenario.

The rest of the paper is organized as follows. In Section 2, we conduct a literature review. In Section 3, we describe the model with single entrant. In Section 4, we characterizes the optimal mechanism when the designer can dictate the winning entrant's production decision (i.e., the partial control scenario). In Section 5, we characterize the optimal mechanism when the designer cannot dictate the winning entrant's production decision (i.e., no control scenario). In Section 6, we extend the model to allow for multiple entrants. In Section 7, we conclude.

2 More related literature

Our paper is primary motivated by the vast literature on auctions with aftermarket competitions. Das Varma [7] examines first-price auctions for a cost reducing innovation among oligopolists who will take part in aftermarket competition. The firms are privately informed about the amount of their production costs that the innovation can reduce. When the aftermarket competition is in Cournot style, there is a unique fully separating equilibrium; if it is in Bertrand style, a fully separating equilibrium may fail to exist. Goeree [10] examines first-price, second-price, and English auctions with abstract aftermarket competitions and compares their revenues. Scarpatetti and Wasser [27] allow multiple objects to be sold in the auctions. In Katzman and Rhodes-Kropf [15], what is allocated through the auction is the access to a duopoly with an incumbent firm.[5] The above strand of literature examines certain specific games and characterizes the corresponding equilibria.

One important issue of the above strand of literature is how much information (regarding the bidders' bids) the auctioneer should reveal after the auction. The common assumption in the above studies is that the auctioneer announces only the transaction price (i.e., the highest bid in a first-price auction, or the second highest bid in a second-price or English auction). Chattopadhyay and Chatterjee [6] and Fan et.al [8] consider the implications of different information revelation rules. In theory, the auctioneer has many options. She can conceal all the information, reveal all the information, reveal information stochastically or partially, etc. Obviously, it is almost impossible to formulate all possible announcement rules one by one. Our paper takes the natural next step to design an optimal mechanism in the presence of possible aftermarket competitions. Such an

[5] There is also a strand of literature which focuses on the case of complete information. This includes Kamien et. al [13] and Katz and Shapiro [14]. Other studies pioneered by Jehiel and Moldovanu [11] assume that types are automatically revealed after the auctions.

approach enables us to characterizes the optimal information revelation rule among all possible ones. We find that in the optimal mechanism, full information revelation is the rule. This is a positive property since sometimes it may be hard for auctioneers to conceal information or to prohibit communications among bidders.[6] Our result shows that there is no need to hide this information as revealing all the information is optimal.

The only paper that studies the aftermarket competition issue from the mechanism design approach is Molnar and Virag [22], the working paper version of Molnar and Virag [23].[7] They consider the optimal auction design problem in an environment where two firms bid to merge with a third firm to reduce marginal costs to gain advantages in the aftermarket competition. Besides the differences in the model structure, their approach is quite different from ours. In their paper, instead of applying the revelation principle, they allow the seller to send signals to bidders after they report their types in the auctions and impose some restrictions on the signals. First, the signal has to be public; second, the loser's information about the winner is assumed to be common knowledge. As they pointed out in the paper, this assumption may be with loss of generality. Indeed, Fan et.al [8] compare the performance of three rules (no disclosure, partial disclosure, and full disclosure), and show that under certain conditions, the optimal rule results in non-common knowledge of the loser's information about the winner. In contrast, we utilize the generalized revelation principle developed in Myerson [24], which accommodates hidden information, hidden action and multiple agents, and obtain an optimal mechanism among all feasible mechanisms. In addition, they focus on uniform distributions, while we allow general distributions.

Our paper is related to the recent literature on Bayesian persuasion pioneered by Kamenica and Gentzkow [12]. They consider the environment with a single Sender and a single Receiver. The Sender can commit to an informative signal about the state of the nature which is initially unknown to everyone. The Receiver takes an action after updating his belief about the state of nature upon observing the signal realization. What make the information disclosure a Bayesian persuasion is the assumption that the Sender cannot distort or conceal information once the signal is realized, which is the feature the designer needs to obey in designing the optimal information structure in our model. In this literature, the Sender designs the signal only and does not take other actions. Therefore, our paper integrates Bayesian persuasion into mechanism design.[8]

Our paper is related to the vast literature on regulation pioneered by Baron and Myerson [2] who consider the optimal way to regulate a monopoly with private production cost. Their analysis has been extended in various directions. For example, Blackorby and Szalay [3] extends the model to accommodate two dimensional private information (production cost and capacity). Auriol and Laffont [1] compare a regulated monopoly with a duopoly. The regulation literature usually focuses on the role of private information. There is no hidden action problem as firms' decisions are fully controlled by the regulator. While their models are applicable in many environments, imperfect

[6] For example, VoiceStream Wireless may be required by law to announce the purchasing price.

[7] Molnar and Virag [23] characterize the revenue maximizing auction when bidders' profitability is additive separable between his true type and his expected perceived type, and the functional function is exogenous given.

[8] The theory in Kamenica and Gentzkow [12] is then extended in several directions. Kamenica and Gentzkow [9] allow multiple senders and investigate whether competitions among senders will lead to more information to be revealed. The general theory is applied to study information disclosure issue in specific settings such as voting in Wang [28] and contest in Zhang and Zhou [30].

regulations may arise due to high monitor costs and lack of essential information. Introducing hidden actions to these models is technically challenging.

3 The model

A revenue maximizing risk neutral designer has one unit of indivisible object to sell in the primary market. There are I risk neutral potential entrants who are interested in acquiring the object. For notational simplicity, we assume that there is a single potential entrant, i.e., $I = 1$. We will illustrate how to generalize the model to any number of potential entrants in Section 5. After the primary market concludes, if the entrant does not obtain the object and stays out, the incumbent behaves as a monopoly in the aftermarket. Otherwise, the entrant and an incumbent compete with each other by choosing their production levels simultaneously as in a Cournot model with homogenous product.

In the aftermarket, the inverse demand function for this product is characterized by a linear function $p = a - q$, where p denotes the market price, q denotes the total supply, and parameter a is a measure of the market size. All firms have constant marginal production costs and zero fixed costs. The entrant's production cost C_E follows a distribution with *c.d.f.* $F_E(c_E)$, *p.d.f.* $f_E(c_E)$, and normalized support $\mathcal{C}_E = [0, 1]$. This C_E is the private information of the entrant. Let c_E denote a realization of C_E. As is common in the literature, we assume that the reverse hazard rate function, $\frac{f_E(c_E)}{F_E(c_E)}$, is strictly decreasing to simplify the characterization of the optimal mechanism. We assume that the incumbent's production cost is commonly known as c_I.[9] In addition, we assume that the market size is relative large so that both the incumbent and the entrant always produce a positive amount in the aftermarket in equilibrium. More specifically, we assume

Assumption 1 $a > 3 \max\{1, c_I\}$.

The entrant's payoff is equal to its profit from the aftermarket minus the payment to the designer. The incumbent's payoff is equal to his profit in the aftermarket. The designer can only collect payment from the entrant but not from the incumbent. Without loss of generality, we normalize the designer's reservation value of the object to zero.

The designer does not have perfect control over the aftermarket. For instance, the designer cannot dictate the production level for the incumbent. Regarding the designer's cotrol power over the entrant's production level in the aftermarket, we focus on two different scenarios. In the first scenario, the designer can dictate the entrant's production level in the aftermarket. In the second scenario, the designer cannot do so. We call the first scenario *partial control* since the designer has control of the entrant's production level but not the incumbent's, and the second one *no control* since the designer cannot control the aftermarket production levels at all.[10] Although the designer

[9] We will discuss how our analysis can be extended to the situation when c_I is the incumbent's private information in the conclusion.

[10] We can also study the case of full control. But it is less interesting, as it is similar to the standard literature on regulation.

cannot fully control the productions levels in the aftermarket directly, she can nevertheless influence its outcome by revealing certain information obtained in the primary market to the aftermarket. This changes the beliefs of the entrant and the incumbent about each other, and therefore, affects their decisions in the aftermarket. As usually the case in practice, we assume that the incumbent can observe whether the entrant enters or not after the primary market concludes.[11] As a result, one important and non-conventional instrument for the designers is determining the optimal information structure in the aftermarket to influent its outcome, which is also the technical challenge lies in. We begin with the partial control scenario.

4 Partial control scenario

In the partial control scenario, the designer decides whether to allocate the object to the entrant, the payment from the entrant, the level of production for the entrant and the amount of information to be revealed to the aftermarket. She can neither ask for payments from the incumbent nor dictate the incumbent's production level in the aftermarket. Formally, the designer offers a mechanism $\mathcal{M} \to R \times \Delta(\{0,1\} \times R_+ \times \Sigma)$ such that, when the entrant reports a message $m \in \mathcal{M}$, he pays $t_E(m) \in R$ and with density $\psi(x, q_E, \sigma)$ the following happen: the entrant either enters ($x = 1$) or stays out ($x = 0$), and the designer dictates a production level $q_E \in R_+$ for the entrant when he enters and reveals certain information $\sigma \in \Sigma$ to the aftermarket.[12]

We make use of the revelation principle developed in Myerson [24] throughout our analysis and restrict our search of the optimal mechanism to direct mechanisms without loss of generality. This revelation principle originally deals with discrete types, but can be extended to continuous types by changing summations to integrals in the derivations.[13] We can thus replicate the outcome induced by any indirect mechanism through a direct mechanism, where the message space is the type space and the information is transmitted through recommendations on actions that are not controlled by the designer. Specifically, a direct mechanism $\mathcal{C}_E \to R \times \Delta(\{0,1\} \times R_+ \times R_+)$ is such that when the entrant reports his production cost $c_E \in \mathcal{C}_E$ to the designer, he pays $t_E(c_E) \in R$ and with density $\pi(x, q_E, q_I|c_E)$ the following happen: the entrant either enters ($x = 1$) or stays out ($x = 0$), and the designer dictates a production level $q_E \in R_+$ for the entrant and sends a private recommendation about the production level $q_I \in R_+$ to the incumbent for the continuation where the entrant enters.[14] Since x, q_E, q_I all depend on c_E, they all convey information about c_E. The market price is derived from the inverse demand function upon the realizations of the total outputs.

The designer chooses a *stochastic* mechanism $(\pi(x, q_E, q_I|c_E), t_E(c_E))$ to maximize her revenue

[11] With a slightly notational changes, we can show that the optimal mechanism in our paper is also optimal when the incumbent cannot observe the entrant's entry.

[12] Since the entrant has quasi-linear preferences, monetary transfers matter only in terms of expectation. Indeed, $t_E(m)$ can always be treated as the expected transfer.

[13] This is similar to Calzolari and Pavan [5] and Zhang and Wang [29], where the revelation principle is utilized to analyze models with resale.

[14] Potentially, recommendations should also be made when the entrant stays out since we assume that the incumbent can observe whether the entrant enters or not. However, when the entrant stays out, the only incentive compatible recommendation for the incumbent is to produce his monopoly level $\frac{a-c_I}{2}$. As a result, we do not need to include this contingent recommendation in the mechanism.

subject to a set of feasibility constraints. Note that the incumbent does not have private information but does have private action, while the entrant has private information but no private action. As a result, the incentive compatibility constraint for the incumbent (IC_I) requires that, given that the entrant truthfully reports his cost in the primary market and follows the designer's dictation in the aftermarket, the incumbent will be obedient and follow the recommendations in the aftermarket. The incentive compatibility constraint for the entrant (IC_E) requires that, given that the incumbent follows the recommendations and the entrant follows the designer's dictation in the aftermarket, the entrant will report his cost truthfully in the primary market.

The participation constraint for the entrant (PC_E) requires that participating in the mechanism is better than the outside option, which is normalized to zero. There is no need to consider the participation constraint for the incumbent, since the designer can neither dictate the production level nor collect any money from him. In fact, when the incumbent receives the recommendation from the designer, he can always choose to ignore this information. As a result, the incumbent's participation constraint is always satisfied. Finally, $\pi(x, q_E, q_I|c_E)$ must be a valid probability distribution: $\forall q_E \in R_+, q_I \in R_+, x \in \{0,1\}, c_E \in \mathcal{C}_E$,

$$\pi(x, q_E, q_I|c_E) \geq 0 \; and \; \int_{R_+} \int_{R_+} \sum_{x=0}^{1} \pi(x, q_E, q_I|c_E) = 1. \tag{1}$$

The designer needs to maximize her revenue, i.e., the expected monetary transfers from the entrant, subject to feasibility constraints IC_I, IC_E, PC_E and (1). In the following subsections, we will examine these constraints one by one, starting backward from the aftermarket. The equilibrium concept we employ is perfect Bayesian Nash equilibria.

4.1 The aftermarket: establishing IC_I

Consider the on-the-equilibrium path continuation games where the entrant has reported his production cost c_E truthfully, and the designer carries out his commitment to implement the mechanism $(\pi(x, q_E, q_I|c_E), t_E(c_E))$. We know that for the aftermarket continuation where the entrant stays out, the incumbent simply produces his monopoly level $\frac{a-c_I}{2}$. What remains is the aftermarket continuation where the entrant enters. Since the entrant's production level is dictated by the designer, we only need to examine the incumbent's incentive compatibility constraint in the aftermarket, i.e., IC_I. When the incumbent receives recommendation q_I, the incumbent needs to choose a production level $\tilde{q}_I$ to maximize his expected profit, i.e.,

$$\max_{\tilde{q}_I \geq 0} \int_{\mathcal{C}_E} \int_{R_+} \Big\{ [a - \tilde{q}_I - q_E - c_I] \, \tilde{q}_I \Big\} \pi(x = 1, q_E, q_I|c_E) f_E(c_E) dq_E dc_E \tag{2}$$

There are two types of uncertainty in the incumbent's payoff. First, the incumbent does not know c_E; second, conditional on c_E, the incumbent does not know the realization of the entrant's dictated production level q_E. As a result, the incumbent needs to form a belief. The information the incumbent has is that the entrant enters $x = 1$ and he receives recommendation q_I. The objective function (2) is strictly concave in $\tilde{q}_I$, and therefore, there exists a unique maximum. IC_I

implies that the incumbent should obey the designer's recommendation, i.e., $\tilde{q}_I = q_I$, and the FOC yields the necessary and sufficient condition for IC_I. Assumption A1 allows us to focus on interior solution throughout the paper.[15] Thus, we obtain the following lemma,

Lemma 1 *IC_I is satisfied if and only if, $\forall q_I$,*

$$q_I = \frac{a - c_I - \int_{\mathcal{C}_E} \int_{R_+} q_E \pi(x = 1, q_E, q_I | c_E) dq_E dF_E(c_E)}{2}$$

4.2 The primary market: establishing IC_E and PC_E

Now we examine the primary market. Note that only the entrant has private information and he is the only one who needs to report , i.e., IC_E. When the entrant reports $\tilde{c}_E$, the designer will implement mechanism $(\pi(x, q_E, q_I|\tilde{c}_E), t_E(\tilde{c}_E))$. The entrant anticipates that the incumbent will be obedient in the aftermarket. Note that the entrant's production level is dictated by the designer in the aftermarket and that the entrant earns a positive profit only when he enters, i.e., $x = 1$. Therefore, knowing his true cost c_E, the entrant's payoff by reporting $\tilde{c}_E$ is given by

$$\begin{aligned} & U_E(c_E, \tilde{c}_E) \qquad (3) \\ = & \int_{R_+} \int_{R_+} \{[a - q_I - q_E - c_E] \, q_E \pi(x = 1, q_E, q_I|\tilde{c}_E)\} \, dq_E dq_I - t_E(\tilde{c}_E). \end{aligned}$$

The expectation is taken because the entrant does not know the realizations of q_E and q_I when he reports in the primary market. The incentive compatibility constraint IC_E and participation constraints PC_E imply that

$$U_E(c_E, \tilde{c}_E) \leq U_E(c_E, c_E), \forall c_E, \tilde{c}_E \qquad (4)$$

$$U_E(c_E, c_E) \geq 0, \forall c_E \qquad (5)$$

As is common in the mechanism design literature, the envelope theorem yields

$$\frac{dU_E(c_E, c_E)}{dc_E} = -\int_{R_+} \int_{R_+} q_E \pi(x = 1, q_E, q_I|c_E) dq_E dq_I$$

$$\Rightarrow \quad U_E(c_E, c_E) = \int_{c_E}^{1} \int_{R_+} \int_{R_+} q_E \pi(x = 1, q_E, q_I|\xi) dq_E dq_I d\xi + U_E(1, 1) \qquad (6)$$

The following lemma shows that IC_E and PC_E are equivalent to the following conditions. The proof is standard and thus omitted.

Lemma 2 *IC_E and PC_E are satisfied if and only if the following conditions hold. $\forall c_E \in \mathcal{C}_E$,*

[15] We will verify at the end that all the production levels in equilibrium are indeed strictly postive.

$$\begin{aligned} t_E(c_E) &= \int_{R_+}\int_{R_+}[a-q_I-q_E-c_E]q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \\ &\quad -\int_{c_E}^{1}\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|\xi)dq_Edq_Id\xi - U_E(1,1), \end{aligned} \tag{7}$$

$$\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \textit{ is decreasing in } c_E$$

$$U_E(1,1)\geq 0$$

The first condition simply rewrites the envelop condition (6), the second one is the monotonicity condition, and the third one is directly from PC_E with $c_E=1$.

4.3 The designer's problem

Lemma 1 characterizes the equivalent conditions for IC_I; Lemma 2 characterizes the equivalent conditions for IC_E and PC_E. As a result, we can rewrite the designer's problem equivalently as

$$\max_{\pi(x,q_E,q_I|c_E),t_E(c_E)} \int_{\mathcal{C}_E} t_E(c_E)dF_E(c_E)$$

subject to:

$$q_I = \frac{a-c_I-\int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_EdF_E(c_E)}{2}, \tag{8}$$

$$t_E(c_E) = \int_{R_+}\int_{R_+}[a-q_I-q_E-c_E]q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \tag{9}$$

$$-\int_{c_E}^{1}\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|\xi)dq_Edq_Id\xi - U_E(1,1), \tag{10}$$

$$\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_Edq_I \textit{ is decreasing in } c_E \tag{11}$$

$$U_E(1,1)\geq 0 \tag{12}$$

$$(1) \tag{13}$$

Although we are able to fully characterize the feasible set, the choice of π is quite complicated as it is a general distribution function and the pointwise maximization cannot be applied directly. Our approach is to derive a tight upper bound revenue and then show that a feasible mechanism can always be constructed to achieve it. As is common in the literature, it is obvious that $U_E(1,1)$ should be set to zero. Substituting (8) and (9) into the objective function yields

$$
\begin{aligned}
& R_P \\
= & \int_{\mathcal{C}_E} t_E(c_E) dF_E(c_E) \\
= & \int_{\mathcal{C}_E} \left\{ \begin{array}{l} \int_{R_+}\int_{R_+}[a-q_I-q_E-c_E]q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I \\ -\int_{c_E}^1\int_{R_+}\int_{R_+} q_E\pi(x=1,q_E,q_I|\xi)dq_E dq_I d\xi, \end{array} \right\} dF_E(c_E) \quad (by\ Eqn.\ (9)) \\
= & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}[a-q_I-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}]q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \\
= & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+} \left\{ \begin{array}{l} \left[\begin{array}{l} a-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}- \\ \frac{a-c_I-\int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E)}{2} \end{array} \right] \times \\ q_E\pi(x=1,q_E,q_I) \end{array} \right\} dq_E dq_I dF_E(c_E) \quad (by\ Eqn.\ (8)) \\
= & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \\
& +\frac{1}{2}\int_{R_+}\int_{\mathcal{C}_E}\int_{R_+} \left\{ \begin{array}{l} \int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E)\times \\ q_E\pi(x=1,q_E,q_I|c_E) \end{array} \right\} dq_E dF_E(c_E) dq_I \\
= & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \\
& +\frac{1}{2}\int_{R_+} \left\{ \begin{array}{l} \int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E) \\ \int_{\mathcal{C}_E}\int_{R_+} q_E\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E) \end{array} \right\} dq_I \\
\leq & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-q_E-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \\
& +\frac{1}{2}\int_{R_+} \left\{ \int_{\mathcal{C}_E}\int_{R_+} q_E^2\pi(x=1,q_E,q_I|c_E)dq_E dF_E(c_E) \right\} dq_I \\
= & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left[\frac{a}{2}+\frac{c_I}{2}-\frac{q_E}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right] q_E\pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \qquad (14)
\end{aligned}
$$

The right hand side of the inequality corresponds to the situation where the entrant's production cost is fully revealed to the incumbent. For the right hand side of (14), point-wise maximization can be applied. Conditional on c_E, let us first consider the term

$$\left[\frac{a}{2}+\frac{c_I}{2}-\frac{q_E}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right]q_E$$

Since $q_E \geq 0$, this term is maximized by setting

$$q_E = \max\{\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)},0\}$$

As a result,

$$\begin{aligned}
& R_P \\
\leq\ & \int_{\mathcal{C}_E}\int_{R_+}\int_{R_+}\left(\max\left\{\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)},0\right\}\right)^2 \pi(x=1,q_E,q_I|c_E)dq_E dq_I dF_E(c_E) \\
=\ & \int_{\mathcal{C}_E}\left[\left(\max\left\{\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)},0\right\}\right)^2 \int_{R_+}\int_{R_+}\pi(x=1,q_E,q_I|c_E)dq_E dq_I\right]dF_E(c_E) \\
\leq\ & \int_{\mathcal{C}_E}\left[\left(\max\left\{\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)},0\right\}\right)^2 \int_{R_+}\int_{R_+}\sum_{x=0}^{1}\pi(x,q_E,q_I|c_E)dq_E dq_I\right]dF_E(c_E) \\
=\ & \int_{\mathcal{C}_E}\left(\max\left\{\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)},0\right\}\right)^2 dF_E(c_E) \quad (by\ Eqn.\ (1)) \qquad (15)
\end{aligned}$$

Note that the term $\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}$ is strictly decreasing in c_E, and is strictly positive at $c_E=0$. We let $\hat{c}_E$ be the point that the above term crosses zero if $\frac{a}{2}+\frac{c_I}{2}-1-\frac{1}{f_E(1)}<0$, and be 1 if $\frac{a}{2}+\frac{c_I}{2}-1-\frac{1}{f_E(1)}\geq 0$.

Define

$$J_E(c_E)=\left[\frac{a}{2}+\frac{c_I}{2}-c_E-\frac{F_E(c_E)}{f_E(c_E)}\right]^2$$

It measures the marginal revenue of selling the object to the entrant and fully revealing the entrant's production cost to the incumbent. Thus,

$$R_P \leq \int_0^{\hat{c}_E} J_E(c_E)dF_E(c_E)$$

As a result, we have established an upper bound revenue for the designer. If we can construct a feasible mechanism that achieves this upper bound revenue, then it will be an optimal mechanism. The following proposition shows that this upper bound revenue is always achievable.

Proposition 1 *In the partial control scenario, the following mechanism maximizes the designer's expected revenue.*

(i) Allocation rule and production level dictation for the entrant upon entry:

$$x = \begin{cases} 1, & if\ 0 \le c_E \le \hat{c}_E; \\ 0, & if\ \hat{c}_E < c_E \le 1; \end{cases} \tag{16}$$

$$q_E = \frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)} \tag{17}$$

(ii) Aftermarket production recommendation for the incumbent when the entrant enters:

$$q_I = \frac{a - 3c_I + 2c_E + \frac{2F_E(c_E)}{f_E(c_E)}}{4} \tag{18}$$

(iii) The entrant's transfer payment to the designer:

$$t_E(c_E) = \begin{cases} \left[\frac{a}{2} + \frac{c_I}{2} - c_E - \frac{F_E(c_E)}{f_E(c_E)}\right]^2 - \int_{c_E}^{\hat{c}_E} \left[\frac{a}{2} + \frac{c_I}{2} - \xi - \frac{F_E(\xi)}{f_E(\xi)}\right] d\xi & if\ 0 \le c_E \le \hat{c}_E \\ 0 & if\ \hat{c}_E \le c_E \le 1 \end{cases}, \tag{19}$$

(iv) The designer's revenue:

$$R_P = \int_0^{\hat{c}_E} J_E(c_E) dF_E(c_E) \tag{20}$$

Proof: It is easy to verify that the above mechanism generates the upper bound revenue. We only need to prove that it satisfies the feasibility constraints. Since the aftermarket production recommendation for the incumbent is a strictly decreasing function of c_E, the incumbent will infer exactly the entrant's production cost when the entrant enters. Thus, it is straightforward to verify that IC_I is satisfied. According to Lemma 2, for IC_E and PC_E, we only need to verify the monotonicity condition (11), which is trivially satisfied. Therefore, the proposed mechanism is feasible, and this completes the proof. **Q.E.D.**

There are many properties for this optimal mechanism, which are summarized in the following corollaries. First, whether to allocate the object to the entrant is a cutoff rule, and the production levels in the aftermarket are in pure strategies. We have,

Corollary 1 *The constructed optimal mechanism is deterministic.*

The above corollary suggests that it is without loss of generality to focus on deterministic mechanisms in search of the optimal mechanism. This is a useful observation since, as to be shown in the no control scenario, it is easier and more straightforward to lay out the model and the analysis with deterministic mechanisms, which suggests that solving the optimal deterministic mechanisms will be a very useful first step in tackling similar problems.

Now, we examine the cutoff $\hat{c}_E$ and obtain some comparative statics.

Corollary 2 *Entry happens more often when the market size is larger or when the incumbent's production cost is higher.*

Note that the designer can raise money only through the entrant. When a or c_I is larger, there are more profits to extract or the entrant is in a more advantageous position in the aftermarket. Therefore, the designer is willing to let the entrant to enter more often.

If we examine Eqn. (18), it reveals that the recommendation to the incumbent is a one-to-one mapping function of the entrant's production cost. This implies that after the incumbent learns the recommendation from the designer, he will infer exactly the entrant's production cost when the entrant enters. We thus conclude

Corollary 3 *In the constructed optimal mechanism, the designer fully reveals the entrant's production cost to the incumbent.*

The intuition is as follows. In the aftermarket competition, strategies are substitute in the Cournot competition. Therefore, the entrant has incentives to signal a lower cost to the incumbent. Since the designer can only collect payment from the entrant and this payment is decreasing in the entrant's production cost, the signaling effect benefits the designer. As a result, it is the best for the designer to create the most signaling effect with the most information disclosure.

In the partial control scenario, the design can dictate the production level for the entrant and act on behalf of the entrant. Since the designer moves first in the primary market and the incumbent chooses its production level later in the aftermarket, this sounds like a Stackelberg competition between the designer (the leader) and the incumbent (the follower). In fact, if we examine the aftermarket production levels upon entry, i.e., (17) and (18), this is close but not precise. There are two differences. First, although the designer controls the entrant's production level in the aftermarket, the entrant has private information. Therefore, the designer has to leave some informational rent to the entrant, which is why the leader's cost is $c_E + F_E(c_E)/f(c_E)$ instead of c_E. Second, after the designer elicits information from the entrant, she can commit whether to disclose this information to the incumbent. Therefore, before the Stackleberg competition takes place, the leader first needs to decide how to disclose his private information to the follower. One may think that the private cost of the leader is irrelevant, since what matters is the leader's production level. But remember that in our model, potentially, the leader can choose not to reveal his own production level to the follower. It only happens that the optimal thing to do is to let the follower to observe the leader's production level. We thus have the following result.

Corollary 4 *In the constructed optimal mechanism, the outcome in the aftermarket is the same as that in a modified Stackelber competition between the designer (the leader) and the incumbent (the follower) under complete information.*

Now, let us discuss the implementation of the optimal mechanism by some indirect mechanism. Note that the monetary transfer function is a strictly decreasing function of c_E and can fully reveal the entrant's private cost. As a result, upon seeing the transaction price for the object, the incumbent can infer exactly the incumbent's production cost. Thus, we have

Corollary 5 *To implement the optimal mechanism in practice, the designer only needs to announce the transaction price for the object to the aftermarket, and does not need to make production recommendations.*

5 No control scenario

In the no control scenario, the designer decides whether to allocate the object to the entrant and the payment, and how much information to reveal to the aftermarket; she can neither ask for payments from the incumbent nor the dictate production level for the incumbent. Furthermore, in contrast to the *partial control* scenario, she cannot dictate the production level for the entrant in the aftermarket either. Again, the revelation principle allows us to focus on direct mechanisms. As shown in the *partial control* scenario the optimal mechanism is deterministic. This also holds in the no control scenario. Instead of going through the general stochastic mechanisms, we will execute our analysis by focusing on the deterministic mechanisms, which provides a much easier and more intuitive way to lay out the model. With deterministic mechanisms, the only uncertainty in the model is the entrant's production cost and we can focus on its updating.

Formally, a direct deterministic mechanism $\mathcal{C}_E \to R \times \{0,1\} \times R_+^2$ is such that when the entrant reports his cost $c_E \in \mathcal{C}_E$ to the designer, the entrant pays $t_E(c_E) \in R$, either enters or stays out $x_E(c_E) \in \{0,1\}$, and the designer sends private recommendations about the production levels $q_E(c_E) \in R_+$ and $q_I(c_E) \in R_+$ to the entrant and the incumbent in the Cournot competition upon the entrant's entry, respectively. Since the entry outcome and recommendations depend on c_E, they are also signals of the entrant's production cost.

The designer chooses a mechanism $(x_E(c_E), q_E(c_E), q_I(c_E), t_E(c_E))$ to maximize her revenue subject to a set of feasibility constraints. Note that the incumbent does not have private information. The incentive compatibility constraint for the incumbent is only for the aftermarket (IC_I^A). It requires that, given that the entrant truthfully reports his cost in the primary market and follows the recommendation in the aftermarket, the incumbent will be obedient and follow the recommendation in the aftermarket. The incentive compatibility constraint for the entrant requires that, given that the incumbent follows the recommendation in the aftermarket, the entrant will report his cost truthfully in the primary market and be obedient in the aftermarket. We can break the entrant's incentive compatibility constraints into two parts. The first part (IC_E^A) is that, if the entrant has truthfully reported his cost in the primary market, it is optimal for him to follow the designer's recommendation in the aftermarket. The second part (IC_E^P) is that, the entrant will truthfully report his cost in the primary market given that he will behave optimally in the aftermarket. The participation constraint for the entrant (PC_E) is the same as in the partial control scenario. For the same reason, there is no participation constraint for the incumbent. Finally, there is only one unit of object:

$$x_E(c_E) \in \{0, 1\} \tag{21}$$

The designer needs to maximize her revenue, i.e., the monetary transfers from the entrant, subject to the feasibility constraints IC_I^A, IC_E^P, IC_E^A, PC_E and (21).

In the following subsections, we will examine these constraints one by one, starting backward from the aftermarket. The equilibrium concept we employ is perfect Bayesian Nash equilibrium. The no control scenario is technically more challenging than the partial control scenario. This is because the entrant has more ways to deviate by misreporting in the primary market and disobeying the recommendation from the designer in the aftermarket at the same time.

5.1 The aftermarket

Let us now examine the incentive compatibility constraint in the aftermarket. For the aftermarket continuation where the entrant stays out, the incumbent will produce the monopoly level $\frac{a-c_I}{2}$. For the aftermarket continuation where the entrant enters, the entrant and the incumbent engage in Cournot competition. The following analysis applies only for the c_E such that $x_E(c_E) = 1$.

5.1.1 The on-equilibrium-path continuation game upon entry: establishing IC_I^A and IC_E^A

First consider the incumbent's incentive compatibility constraint in the aftermarket, i.e., IC_E^I. Along the on-the-equilibrium-path continuation game where the entrant reports his production cost c_E truthfully and enters, the designer carries out his commitment and implements the mechanism $(q_E(c_E), q_I(c_E), t_E(c_E))$. With deterministic mechanisms, from the prospective of the incumbent, the uncertainty in his payoff in the Cournot competition only depends on the entrant's production cost. Let $\mathcal{Q}_I = \{q_I(c_E) : x_E(c_E) = 1\}$ denote the set of all possible equilibrium recommendations in the Cournot competition for the incumbent when the entrant enters, and let Q_I denote the associated random variable. Suppose that the incumbent receives a recommendation $q_I \in \mathcal{Q}_I$. Note that potentially many different values of c_E could lead to the recommendation q_I. We can formulate the incumbent's maximization problem as:

$$\max_{\tilde{q}_I \geq 0} \mathbf{E} \left\{ [a - \tilde{q}_I - q_E(C_E) - c_I]\, \tilde{q}_I \middle| Q_I = q_I \right\}$$

The expectation is taken on C_E. Since the objective function is strictly concave, there exists a unique maximum. In equilibrium, the incumbent should obey the designer's recommendation, i.e., $\tilde{q}_I = q_I$, and the FOC yields the obedient condition for the incumbent: $\forall q_I \in \mathcal{Q}_I$,[16]

$$q_I = \frac{a - c_I - \mathbf{E}\{q_E(C_E)|Q_I = q_I\}}{2} \tag{22}$$

[16] Again, Assumption 1 guarantees interior solutions in the optimal mechanism.

Now consider the entrant's incentive compatibility constraint in the aftermarket upon entry, i.e., IC_E^A. Suppose that the entrant with production cost c_E, such that $x_E(c_E) = 1$, has truthfully reported to the designer. Then he anticipates that the incumbent will obey the designer's recommendation to produce $q_I(c_E)$ in the Cournot competition. With deterministic mechanisms, there is no uncertainty in the entrant's payoff since he knows c_E. As a result, when the entrant receives recommendation $q_E(c_E)$, he maximizes his expected payoff:

$$\max_{\tilde{q}_E \geq 0} [a - q_I(c_E) - \tilde{q}_E - c_E]\, \tilde{q}_E$$

Note that the objective function is strictly concave, and therefore, there exist a unique maximum. In equilibrium, the entrant should obey the designer's recommendation, i.e. $\tilde{q}_E = q_E(c_E)$, and the FOC yields the obedient condition for the entrant:

$$q_E(c_E) = \frac{a - c_E - q_I(c_E)}{2}. \tag{23}$$

From (22) and (23), we can solve the incentive compatible recommendations for each firm in the following lemma.

Lemma 3 *IC_I^A and IC_E^A are satisfied, if and only if $\forall c_E$ such that $x_E(c_E) = 1$ and $q_I(c_E) = q_I$, we have*

$$q_I = \frac{a}{3} - \frac{2}{3}c_I + \frac{\mathbf{E}\{C_E|Q_I = q_I\}}{3}$$

$$q_E(c_E) = \frac{a}{3} + \frac{1}{3}c_I - \frac{c_E}{2} - \frac{\mathbf{E}\{C_E|Q_I = q_I\}}{6},$$

Proof: Substituting (23) into (3) yields

$$\begin{aligned} q_I &= \frac{a - c_I - \mathbf{E}\left\{\frac{a - C_E - q_I(C_E)}{2}|Q_I = q_I\right\}}{2} \\ &= \frac{a - c_I - \frac{a - \mathbf{E}\{C_E|Q_I = q_I\} - q_I}{2}}{2} \\ \Leftrightarrow \quad q_I &= \frac{a}{3} - \frac{2}{3}c_I + \frac{\mathbf{E}\{C_E|Q_I = q_I\}}{3} \end{aligned} \tag{24}$$

Note that (22) and (23) hold for any c_E such that $x_E(c_E) = 1$. Therefore, substituting the above equation with $q_I = q_I(c_E)$ into (23) yields the formula for $q_E(c_E)$. **Q.E.D.**

5.1.2 A deviation

In order to determine the incentive compatibility constraints in the primary market, we need to know when the entrant reports his valuation to be $\tilde{c}_E \neq c_E$ and enters, how he would act in the aftermarket. If $x_E(\tilde{c}_E) = 0$, the aftermarket is irrelevant to him since he will not enter. Otherwise, he knows that the incumbent will obey the designer's recommendation and produce $q_I(\tilde{c}_E)$. Therefore, the entrant's problem in the Cournot competition is given by:

$$\max_{\tilde{q}_E \geq 0} \left[a - q_I(\tilde{c}_E) - \tilde{q}_E - c_E\right] \tilde{q}_E \tag{25}$$

Note that the objective function is strictly concave, and therefore, there exists a unique maximum, and the FOC yields,[17]

$$\tilde{q}_E = \frac{a - q_I(\tilde{c}_E) - c_E}{2} \tag{26}$$

We thus have the following lemma.

Lemma 4 *When the entrant reports $\tilde{c}_E$ in the primary market and enters the aftermarket, he will choose a production level $\frac{a-q_I(\tilde{c}_E)-c_E}{2}$ in the aftermarket.*

5.2 The primary market: Establishing IC_E^P and PC_E

Now we examine the primary market. Note that only the entrant has private information and he is the only one who needs to report (IC_E^P). Given that the incumbent is obedient and the entrant chooses the production level in the competition optimally according to Lemma 4, the entrant's payoff by reporting $\tilde{c}_E$ is

$$\begin{aligned} & U_E(c_E, \tilde{c}_E) \\ = & \left\{\left[a - q_I(\tilde{c}_E) - \frac{a - q_I(\tilde{c}_E) - c_E}{2} - c_E\right] \frac{a - q_I(\tilde{c}_E) - c_E}{2}\right\} x_E(\tilde{c}_E) - t_E(\tilde{c}_E) \\ = & \left[\frac{a - q_I(\tilde{c}_E) - c_E}{2}\right]^2 x_E(\tilde{c}_E) - t_E(\tilde{c}_E). \end{aligned} \tag{27}$$

The incentive compatibility IC_E^P and participation constraints PC_E imply that

$$U_E(c_E, \tilde{c}_E) \leq U_E(c_E, c_E), \forall c_E, \tilde{c}_E \tag{28}$$

$$U_E(c_E, c_E) \geq 0, \forall c_E \tag{29}$$

[17] Assumption 1 guarantees that the solution is interior.

As common in the mechanism design literature, the envelope theorem yields

$$\begin{aligned}\frac{dU_E(c_E,c_E)}{dc_E} &= 2\left[\frac{a-q_I(\tilde{c}_E)-c_E}{2}\right](-\frac{1}{2})x_E(\tilde{c}_E)\Big|_{\tilde{c}_E=c_E} \\ &= -q_E(c_E)x_E(c_E) \\ &\Rightarrow U_E(c_E,c_E)=\int_{c_E}^1 q_E(\xi)x_E(\xi)d\xi+U_E(1,1) \qquad (30)\end{aligned}$$

As a result, IC_E^P and PC_E imply the following lemma.

Lemma 5 *IC_E^P and PC_E are satisfied only if the following conditions hold:*

$$t_E(c_E)=q_E(c_E)^2x_E(c_E)-\int_{c_E}^1 q_E(\xi)x_E(\xi)d\xi-U_E(1,1), \qquad (31)$$

$$U_E(1,1)\geq 0 \qquad (32)$$

The first condition is simply a rewrite of (30), and the second is directly from PC_E with $c_E=1$. In contrast to the standard literature, a necessary and sufficient condition for IC_E^P and PC_E cannot be obtained. This is because the entrant can deviate by misreporting his production cost and disobeying the recommendation at the same time.

5.3 The designer's problem

Lemma 3 characterizes the equivalent conditions for IC_I^A and IC_E^A; Lemma 5 only characterizes the necessary conditions for IC_E^P and PC_E. Our approach is to study a relaxed problem of the original problem and work out the optimal mechanism there. We then prove that this mechanism is also feasible in the original problem and is therefore optimal in the original problem. We can formulate the relaxed problem as follows:

$$\max_{q_I(c_E),q_E(c_E),x_E(c_E),t_E(c_E)}\int_0^1 t_E(c_E)dF_E(c_E)$$

subject to:

$$\begin{aligned}q_E(c_E) &= \frac{a}{3}+\frac{1}{3}c_I-\frac{c_E}{2}-\frac{1}{6}\mathbf{E}\left\{C_E|Q_I=q_I\right\}, &(33)\\ t_E(c_E) &= q_E(c_E)^2x_E(c_E)-\int_{c_E}^1 q_E(\xi)x_E(\xi)d\xi-U_E(1,1), &(34)\\ U_E(1,1) &\geq 0 &(35)\end{aligned}$$

(21)

The reason why this problem is a relaxed problem of the original problem is as follows. First, the objective functions are the same. Second, the feasibility constraints are implied by those in the original problem according to Lemma 3 and Lemma 5, and are therefore less restrictive. As a result, the solution provides an upper bound revenue for the original problem. Again, the above problem cannot be solved by pointwise maximization.

As is common in the literature, it is obvious that $U_E(1,1)$ should be set to zero. Substituting (33) and (34) into the objective function, the designer's problem becomes

$$
\begin{aligned}
& R_N \\
= & \int_0^1 \left\{ q_E(c_E)^2 x_E(c_E) - \int_{c_E}^1 q_E(\xi) x_E(\xi) d\xi \right\} dF_E(c_E) \quad (36) \\
= & \int_0^1 \left\{ \left[q_E(C_E) - \frac{F_E(C_E)}{f_E(C_E)} \right] q_E(C_E) x_E(C_E) \right\} dF_E(c_E) \quad (37) \\
= & \mathbf{E} \left\{ \mathbf{E} \left\{ \left[q_E(C_E) - \frac{F_E(C_E)}{f_E(C_E)} \right] q_E(C_E) \middle| Q_I \right\} \right\} \quad \textit{(The event } x_E(c_E) = 0 \textit{ does not affect the revenue)} \\
= & \mathbf{E} \left\{ \mathbf{E} \left\{ \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} - \tfrac{\mathbf{E}\{C_E|Q_I\}}{6} - \tfrac{F_E(C_E)}{f_E(C_E)} \right] \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} - \tfrac{\mathbf{E}\{C_E|Q_I\}}{6} \right] \middle| Q_I \right\} \right\} \quad \textit{(by Eqn. (33))} \\
= & \mathbf{E} \left\{ \mathbf{E} \left\{ \begin{array}{l} \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} - \tfrac{F_E(C_E)}{f_E(C_E)} \right] \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} \right] \\ - \left[\tfrac{2a}{3} + \tfrac{2}{3} c_I - C_E - \tfrac{F_E(C_E)}{f_E(C_E)} \right] \tfrac{\mathbf{E}\{C_E|Q_I\}}{6} \\ + \tfrac{\mathbf{E}\{C_E|Q_I\}^2}{36} \end{array} \middle| Q_I \right\} \right\} \\
= & \mathbf{E} \left\{ \begin{array}{l} \mathbf{E} \left\{ \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} - \tfrac{F_E(C_E)}{f_E(C_E)} \right] \left[\tfrac{a}{3} + \tfrac{1}{3} c_I - \tfrac{C_E}{2} \right] \middle| Q_I \right\} \\ - \mathbf{E} \left\{ \left[\tfrac{2a}{3} + \tfrac{2}{3} c_I - C_E - \tfrac{F_E(C_E)}{f_E(C_E)} \right] \middle| Q_I \right\} \tfrac{\mathbf{E}\{C_E|Q_I\}}{6} \\ + \tfrac{\mathbf{E}\{C_E|Q_I\}\mathbf{E}\{C_E|Q_I\}}{36} \end{array} \right\} \quad (38)
\end{aligned}
$$

We need the following lemma to proceed further.

Lemma 6 *(Majorization Inequality) Suppose $h'_I(c_E), h'_E(c_E) \geq 0$, then*

$$\mathbf{E}[(h_I(c_E) h_E(c_E)] \geq \mathbf{E}[h_I(c_E)]\mathbf{E}[h_E(c_E)].$$

Therefore, according to the above Majorization Inequality, we obtain

$$\begin{aligned} R_N &\leq \mathbf{E}\left\{ \begin{array}{l} \mathbf{E}\left\{ \left[\frac{a}{3}+\frac{1}{3}c_I-\frac{C_E}{2}-\frac{F_E(C_E)}{f_E(C_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{C_E}{2}\right] \middle| Q_I \right\} \\ -\mathbf{E}\left\{ \left[\frac{2a}{3}+\frac{2}{3}c_I-C_E-\frac{F_E(C_E)}{f_E(C_E)}\right]\frac{C_E}{6} \middle| Q_I \right\} \\ +\mathbf{E}\left\{ \frac{C_E^2}{36} \middle| Q_I \right\} \end{array} \right\} \\ &= \mathbf{E}\left\{\mathbf{E}\left\{ \left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2C_E}{3}-\frac{F_E(C_E)}{f_E(C_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2C_E}{3}\right] \middle| Q_I \right\}\right\} \quad (39) \\ &= \mathbf{E}\left\{ \left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2C_E}{3}-\frac{F_E(C_E)}{f_E(C_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2C_E}{3}\right] x_E(C_E) \right\} \quad (40) \\ &= \int_0^1 \left\{ \left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right] \right\} x_E(c_E)f_E(c_E)dc_E \quad (41) \end{aligned}$$

(42)

The right hand side of the inequality corresponds to the situation where the entrant's production cost is fully revealed to the incumbent. Note that the inequality following for any $x_E(c_E)$, which means regardless of the allocation rule, it is always the best to fully reveal the entrant's production cost to the incumbent. Define

$$\tilde{J}_E(c_E) = \left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}\right]\left[\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}\right] \quad (43)$$

It measures the marginal revenue of issuing the object to the entrant and fully revealing the entrant's production cost to the incumbent. Note that the term $\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}$ is always strictly positive according to Assumption 1. The term $\frac{a}{3}+\frac{1}{3}c_I-\frac{2c_E}{3}-\frac{F_E(c_E)}{f_E(c_E)}$ is strictly decreasing in c_E, and is strictly positive at $c_E = 0$. We let c_E^* be the point that the above term crosses zero if $\frac{a}{3}+\frac{1}{3}c_I-\frac{2}{3}-\frac{1}{f_E(1)} < 0$, and be 1 if $\frac{a}{3}+\frac{1}{3}c_I-\frac{2}{3}-\frac{1}{f_E(1)} \geq 0$. We thus obtain

$$R_N \leq \int_0^{c_E^*} \tilde{J}_E(c_E)dF_E(c_E)$$

As a result, we have established an upper bound revenue for the designer. The following proposition shows that this upper bound revenue is always achievable.

Proposition 2 *The following mechanism maximizes the designer's revenue.*

(i) Allocation rule:

$$x_E(v_E) = \begin{cases} 1, & if\ 0 \le c_E \le c_E^*; \\ 0, & if\ c_E^* < c_E \le 1; \end{cases} \tag{44}$$

(ii) Aftermarket production recommendations in the Cournot competition:

$$q_I(c_E) = \frac{a - 2c_I + c_E}{3} \tag{45}$$

$$q_E(c_E) = \frac{a + c_I - 2c_E}{3} \tag{46}$$

(iii) The entrant's transfer payment to the designer:

$$t_E(c_E) = \begin{cases} \left(\frac{a+c_I-2c_E}{3}\right)^2 - \int_{c_E}^{c_E^*} \left(\frac{a+c_I-2\xi}{3}\right) d\xi, & if\ 0 \le c_E \le c_E^*; \\ 0, & if\ c_E^* < c_E \le 1; \end{cases}, \tag{47}$$

(iv) The designer's revenue:

$$R_N = \int_0^{c_E^*} \tilde{J}_E(c_E) dF_E(c_E)$$

Proof: It is easy to verify that the above mechanism generates the upper bound revenue. We only need to prove that it satisfies the feasibility constraints. Since the aftermarket recommendation $q_I(c_E)$ is a strictly increasing function, upon seeing the recommendation, the incumbent will infer exactly the entrant's production cost. Lemma 3 then confirms that IC_I^A and IC_E^A are satisfied. Now consider IC_E^P and PC_E. When $\tilde{c}_E > c_E^*$,

$$U_E(c_E, \tilde{c}_E) = 0$$

When $\tilde{c}_E < c_E^*$,

$$U_E(c_E, \tilde{c}_E) = \left[\frac{a - \frac{a-2c_I+\tilde{c}_E}{3} - c_E}{2}\right]^2 - \left(\frac{a + c_I - 2\tilde{c}_E}{3}\right)^2 + \int_{\tilde{c}_E}^{c_E^*} \frac{a + c_I - 2\xi}{3} d\xi$$

$$
\begin{aligned}
\frac{\partial U_E(c_E,\tilde{c}_E)}{\partial \tilde{c}_E} &= 2(-\frac{1}{6})\left[\frac{a-\frac{a-2c_I+\tilde{c}_E}{3}-c_E}{2}\right]-2(-\frac{2}{3})\left(\frac{a+c_I-2\tilde{c}_E}{3}\right)-\frac{a+c_I-2\tilde{c}_E}{3} \\
&= (-\frac{1}{3})\left[\frac{2a+2c_I-\tilde{c}_E-3c_E}{6}\right]+\frac{1}{3}\left(\frac{a+c_I-2\tilde{c}_E}{3}\right) \\
&= -\left[\frac{2a+2c_I-\tilde{c}_E-3c_E}{18}\right]+\left(\frac{2a+2c_I-4\tilde{c}_E}{18}\right) \\
&= \left[\frac{-\tilde{c}_E+c_E}{6}\right] \qquad (48)
\end{aligned}
$$

$$
\frac{\partial^2 U_E(c_E,\tilde{c}_E)}{\partial {\tilde{c}_E}^2} = -\frac{1}{6}
$$

Thus, $\tilde{c}_E = c_E$ is a maximum and truthfully reporting is optimal, i.e., IC_E^P is satisfied. PC_E is satisfied since $U_E(1,1) = 0$. **Q.E.D.**

Similarly, we can summarize some properties of the optimal mechanism in some corollaries. Corollaries 1, 2, 3 in the partial control scenario continue to hold. Similar to Corollary 4, we have the following corollary.

Corollary 6 *In the constructed optimal mechanism, the outcome in the aftermarket is the same as that in a Cournot competition under complete information.*

When the designer has no control over the aftermarket, a simple and popular mechanism is to make a take-it-or-leave-it to the entrant. However, such mechanism will not be able to elicit all the information from the entrant. With the take-it-or-leave-it offer, the designer can only infer whether the entrant's production cost is above or below a cutoff instead of its exact value. As a result, we have

Corollary 7 *A take-it-or-leave-it offer can never be revenue maximizing for the designer.*[18]

In both scenarios, entry happens only when the entrant's production cost is lower than a cutoff. If we compare the cutoffs in the two scenario, we obtain $c_E^* \leq \hat{c}_E$, which implies

Corollary 8 *Entry happens more often under partial control than under no control.*

The intuition is that when the designer has more control over the entrant, she can extract more surplus from the entrant, and therefore, she is more willing to let the entrant to produce. Furthermore, if we compare the revenues between the no control scenario and partial control scenario, we obtain

[18] In the partial control scenario, the designer needs to choose the entrant's production level, and it sounds strange to mention take-it-or-leave-it offers. Obviously, a take-it-or-leave-it offer is not optimal in that situation.

Corollary 9 *The revenue is strictly higher under partial control than under no control.*

This is because when the designer has partial control, at least she can implement the same mechanism that is optimal under no control. Comparing (20) and (48), we conclude that they are not equal. It is easy to show that when the designer can also dictate a production level for the incumbent, she can achieve even higher revenue, which is a standard regulation problem. This suggests that the moral hazard problem does limit the designer's rent extraction ability even with risk neutral agents, in contrast to the previous literature such as McAfee and McMillan [21].

6 Extension to I entrants

The restriction to a single entrant is only for expositional simplicity. The analysis can be easily extended to allow I entrants. Here, we focus on the no control scenario, and the partial control scenario is similar. The marginal revenue of allocating the object to entrant i is simply $\tilde{J}_i(c_i)$ by replacing subscript E to i in equation (43). Therefore, the designer simply allocates the object to the entrant with the highest $\tilde{J}_i(c_i)$ if it is positive. The recommendations remain the same and the incumbent will infer the exact production cost of the winning entrant in the aftermarket.

When $I = 1$, we have illustrated that the commonly observe mechanism, i.e., a take-it-or-leave-it offer, is suboptimal. However, when $I \geq 2$, the optimal mechanism can be implemented by a simple and commonly adopted mechanism. Note that $\tilde{J}_i(c_i)$ is strictly decreasing when it is positive. Therefore, if the entrants are symmetric, it is in the designer's interest to allocate the object to the firm with the lowest cost, conditional on it is lower than the cutoff c_i^*. We thus have

Proposition 3 *Under the no control scenario, when there are multiple symmetric entrants, a first-price auction with a reserve price and the announcement of the winning price implements the optimal mechanism. In contrast, English auctions can never be optimal.*

When the entrants adopt symmetric decreasing bidding function in the auction, the entrant with the lowest cost wins. Furthermore, from the transaction price, the incumbent will infer the winning entrant's exact production cost, and the Cournot competition is as if under complete information. However, English auctions can never be optimal since they cannot fully reveal the winner's type. For instance, the transaction price in English auctions only contains information about the second lowest production cost. Therefore, no matter what information is revealed in English auctions, the winner's production cost remains unknown.

7 Discussions and Conclusions

In this paper, we study the optimal mechanism design problem with aftermarket competition in which the designer has perfect control of the primary market but not the aftermarket. We fully characterize the optimal mechanisms for two scenario: partial control and no control. The optimal mechanism are deterministic and the designer fully reveals the winning entrant's production cost to the incumbent under both scenarios. In the no control scenario, if there is a single entrant, it is never optimal for the designer to make a take-it-or-leave-it offer to the entrant. When there are multiple symmetric entrants, while the optimal mechanism can be implemented by a first-price auction with a reserve price and the announcement of the winning bid, English auctions can never be optimal. Entry happens more often and the designer can achieve strictly more revenue in the partial than in the no control scenario. One of the main questions in the auction literature is comparing the performance of standard auctions. Our findings provides lead to the following takeaway for auction designers in practice: When aftermarket interaction plays a role, first-price auctions are better than English auctions since they are more efficient in collecting information from the bidders.

Our model can be easily extended in several directions. First, we can allow c_I to be the incumbent's private information. However, given that the designer can neither elicit information nor collect payments from the incumbent, all that matters is the incumbent's expected production cost and c_I can be reinterpreted accordingly. Second, the objective function of the designer could be different from just maximizing revenue. It can be shown that the mains results remain to hold when the designer also cares about welfare. Third, we assume that the incumbent can observe whether the entrant enters or not after the primary market concludes. It can be shown that our mechanism is also optimal in the opposite case by changing the notations slightly.

The following extensions are less straightforward. First, the aftermarket competition may not need to be Cournot competition. Bertrand or Stackelberg competitions may be applicable for different industries. It would be interesting to extend the model to accommodate a general abstract aftermarket competition or even more general aftermarket interaction such as resale and collusion. While the current paper illustrates that it is optimal to fully reveal information to the aftermarket, one may ask whether this is true for any aftermarket interactions. The answer is negative. In Zhang and Wang [29], the aftermarket interaction is resale, and it is found that fully concealing the information is optimal. It is thus a more subtle question on how to reveal the information to the aftermarket in a general setup. Second, the entire market structure could be modeled differently. For example, both the entrant and the incumbent could interact in the primary market. Third, the regulation literature usually assumes full controlling power by the regulator and no moral hazard problem. It would be interesting to reexamine the same issues with imperfect regulator power in the presence of moral hazard problem. We leave these open questions to future investigations.

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